J. Fluid Mech. (1971), *vol.* 48, *part* 4, *pp.* 763–769
Printed in Great Britain

The effect of velocity sensitivity on temperature derivative statistics in isotropic turbulence

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(Received 13 October 1970)

The velocity sensitivity of a resistance-wire temperature sensor is expressed in terms of sensor parameters, and the resulting errors in temperature derivative moments in isotropic turbulence are evaluated. It is shown that velocity sensitivity of a degree completely negligible for most purposes causes severe contamination of the measured third moment. The contamination terms are shown to be production rates of the mean square temperature gradient and vorticity, respectively, and therefore create positive values of measured derivative skewness. The dominant contamination term is related to the temperature spectrum through the balance equation for the mean-square temperature gradient, and calculations based on an assumed spectral form show that under typical conditions the measured skewness is large. This mechanism could provide an alternative to anisotropy as an explanation of the positive skewnesses recently measured in the atmosphere.

1. Introduction

Thin resistance-wire temperature sensors are often used to measure temperature fluctuations in turbulence. Recent atmospheric experiments (reported by Stewart 1969 and Gibson, Stegen & Williams 1970) with very small wires have given data on temperature derivative statistics, including skewness and the rate of molecular dissipation of temperature fluctuations. Such data are useful in testing, for example, the postulate that the fine structure of temperature fields at large Reynolds number is universal and isotropic. The purpose of this paper is to point out that velocity sensitivity of the sensor can have a strong influence on certain temperature derivative statistics in isotropic turbulence, and that unless the effect is minimized the fine structure will appear to be non-universal and anisotropic.

2. Velocity sensitivity

Velocity sensitivity occurs because a resistance-type sensor operates slightly above the ambient fluid temperature due to the heating effect of the small current it carries. The current is necessary to detect resistance fluctuations, which are proportional to the wire temperature changes caused by temperature fluctuations in the ambient fluid. Since the wire is slightly overheated, its temperature also responds to changes in the heat-transfer coefficient caused by fluid velocity changes. A gust of higher speed constant-temperature fluid increases the rate of heat transfer and decreases the wire temperature; this is interpreted in the instrument output as an ambient temperature decrease. The apparent, or measured, fluctuating temperature signal from a sensor normal to the flow therefore contains an unwanted component due to the streamwise velocity sensitivity:

$$\theta^m = \theta - c u_1. \tag{1}$$

In practice the velocity sensitivity c is kept small by keeping the wire overheat low, and the error in temperature variance is usually quite small. However, not all temperature statistics are measured with comparable accuracy. To investigate the effect on derivatives, we start from

$$\frac{\partial \theta^m}{\partial x_1} = \frac{\partial \theta}{\partial x_1} - c \frac{\partial u_1}{\partial x_1}, \tag{2}$$

which is obtained from (1) by interpreting time derivatives as streamwise (x_1) derivatives through Taylor's hypothesis. Second moments in isotropic turbulence are therefore measured as

$$\overline{\left(\frac{\partial \theta}{\partial x_1}\right)^{2m}} = \overline{\left(\frac{\partial \theta}{\partial x_1}\right)^{2}} + c^2 \overline{\left(\frac{\partial u_1}{\partial x_1}\right)^{2}}. \tag{3}$$

The third moment of $\partial\theta/\partial x_1$ vanishes in isotropic turbulence, but (2) shows that its measured value does not

$$\overline{\left(\frac{\partial \theta}{\partial x_1}\right)^{3m}} = -3c \overline{\frac{\partial \theta}{\partial x_1} \frac{\partial \theta}{\partial x_1} \frac{\partial u_1}{\partial x_1}} - c^3 \overline{\left(\frac{\partial u_1}{\partial x_1}\right)^{3}}. \tag{4}$$

Although the second term on the right of (4) is the more familiar, we will now show that the first term is usually more important. In the process, we will reveal the similar roles they play in the dynamics of turbulent vorticity and temperature gradient fields.

We start from the balance equation for mean-square turbulent vorticity in large Reynolds number, stationary turbulence,

$$\frac{\partial \overline{\omega_i \omega_i}}{\partial t} = 2\overline{\omega_i \omega_j \frac{\partial u_i}{\partial x_j}} - 2\nu \overline{\frac{\partial \omega_i}{\partial x_j} \frac{\partial \omega_i}{\partial x_j}} = 0. \tag{5}$$

This expresses the average balance that exists between the production rate caused by vortex stretching and the viscous destruction rate. The production rate can be written

$$2\overline{\omega_i \omega_j \frac{\partial u_i}{\partial x_j}} = -2 \overline{\frac{\partial u_i}{\partial x_j} \frac{\partial u_i}{\partial x_k} \frac{\partial u_j}{\partial x_k}} = -35 \overline{\left(\frac{\partial u_1}{\partial x_1}\right)^{3}}, \tag{6}$$

by the definition of vorticity and from isotropy. Therefore $-\overline{(\partial u_1/\partial x_1)^3}$ is a production rate of mean-square turbulent vorticity, and is required to be positive; (4) then shows that this contributes to positive values of $\overline{(\partial\theta/\partial x_1)^{3m}}$.

We can make an analogous interpretation of

$$\overline{\frac{\partial \theta}{\partial x_1} \frac{\partial \theta}{\partial x_1} \frac{\partial u_1}{\partial x_1}}$$

by using the balance equation for the mean-square temperature gradient. We start from the fluctuating temperature equation

$$\frac{\partial \theta}{\partial t}+\bar{U}_j\frac{\partial \theta}{\partial x_j}+u_j\frac{\partial \theta}{\partial x_j}-\overline{u_j\frac{\partial \theta}{\partial x_j}}+u_j\frac{\partial \bar{\Theta}}{\partial x_j}=D\frac{\partial^2 \theta}{\partial x_j\,\partial x_j}. \tag{7}$$

Here $\bar{U}_j$ is the mean velocity, $\bar{\Theta}$ is the mean temperature, and D is the thermal diffusivity of the fluid. Differentiating with respect to x_i, multiplying by $2\partial\theta/\partial x_i$, and averaging then gives a balance equation for $\overline{(\partial\theta/\partial x_i)^2}$. In large Reynolds number, stationary turbulence, this looks much like the mean-square vorticity equation,

$$\frac{\partial}{\partial t}\left(\overline{\frac{\partial \theta}{\partial x_i}\frac{\partial \theta}{\partial x_i}}\right)=-2\,\overline{\frac{\partial \theta}{\partial x_i}\frac{\partial \theta}{\partial x_j}\frac{\partial u_i}{\partial x_j}}-2D\,\overline{\frac{\partial^2 \theta}{\partial x_i\partial x_j}\frac{\partial^2 \theta}{\partial x_i\partial x_j}}=0. \tag{8}$$

The third moment in this balance equation is evidently the production rate of $\overline{\partial\theta/\partial x_i\,\partial\theta/\partial x_i}$ caused by the stretching of the temperature field by the turbulent strain field. This is balanced, on the average, by molecular smoothing of the temperature gradient field. Isotropy implies that the production term is

$$-2\,\overline{\frac{\partial \theta}{\partial x_i}\frac{\partial \theta}{\partial x_j}\frac{\partial u_i}{\partial x_j}}=-15\,\overline{\frac{\partial \theta}{\partial x_1}\frac{\partial \theta}{\partial x_1}\frac{\partial u_1}{\partial x_1}} \tag{9}$$

so we can interpret

$$-\overline{\frac{\partial \theta}{\partial x_1}\frac{\partial \theta}{\partial x_1}\frac{\partial u_1}{\partial x_1}}$$

as a production rate of mean-square temperature gradient, and it is positive. Both terms on the right of (4) are therefore positive, and the measured value of $\overline{(\partial\theta/\partial x_1)^3}$ in isotropic turbulence will also be positive.

That

$$\overline{\frac{\partial \theta}{\partial x_1}\frac{\partial \theta}{\partial x_1}\frac{\partial u_1}{\partial x_1}}$$

should be negative can also be seen intuitively.† In regions where $\partial u_1/\partial x_1$ is positive, the temperature field is being stretched, and we expect reduced values of $(\partial\theta/\partial x_1)^2$. Conversely, where $\partial u_1/\partial x_1$ is negative, we expect stronger temperature gradients; these contributions should dominate, making

$$\overline{\frac{\partial \theta}{\partial x_1}\frac{\partial \theta}{\partial x_1}\frac{\partial u_1}{\partial x_1}}$$

negative.

We know therefore that positive skewness of $\partial\theta/\partial x_1$ will be measured in isotropic turbulence, but its magnitude remains to be estimated. The case of most practical importance involves a velocity sensitivity sufficiently small that the error in the temperature gradient variance, as indicated by (3), is negligible; in this case the mixed third moment dominates in (4), and we have

$$\overline{\left(\frac{\partial \theta}{\partial x_1}\right)^{3m}}=-3c\,\overline{\frac{\partial \theta}{\partial x_1}\frac{\partial \theta}{\partial x_1}\frac{\partial u_1}{\partial x_1}}. \tag{10}$$

† The author is indebted to an anonymous referee for this interpretation.

There appear to be no published measurements of

$$\overline{\frac{\partial\theta}{\partial x_1}\frac{\partial\theta}{\partial x_1}\frac{\partial u_1}{\partial x_1}},$$

so we cannot directly evaluate (10). We can, however, turn to (8) and (9) and write

$$\overline{\left(\frac{\partial\theta}{\partial x_1}\right)^{3^m}} = 0{\cdot}4cD\,\overline{\frac{\partial^2\theta}{\partial x_i\,\partial x_j}\frac{\partial^2\theta}{\partial x_i\,\partial x_j}}, \tag{11}$$

which can in turn be written in terms of the temperature spectrum:

$$\overline{\left(\frac{\partial\theta}{\partial x_1}\right)^{3^m}} = 0{\cdot}4cD\iiint_{-\infty}^{\infty}\kappa^4\Phi(\boldsymbol{\kappa})\,d\kappa_1\,d\kappa_2\,d\kappa_3. \tag{12}$$

Here $\Phi(\boldsymbol{\kappa})$ is the temperature spectrum, which has the property

$$\iiint_{-\infty}^{\infty}\Phi(\boldsymbol{\kappa})\,d\kappa_1\,d\kappa_2\,d\kappa_3 = \overline{\theta^2}. \tag{13}$$

Since isotropy implies that Φ can depend only on κ, the magnitude of $\boldsymbol{\kappa}$, we can integrate over spherical shells and write (12) as

$$\overline{\left(\frac{\partial\theta}{\partial x_1}\right)^{3^m}} = 0{\cdot}4cD\int_0^{\infty}\kappa^4F(\kappa)\,d\kappa, \tag{14}$$

where $F(\kappa)$, called the three-dimensional temperature spectrum, is

$$F(\kappa) = 4\pi\kappa^2\Phi(\kappa). \tag{15}$$

Corrsin (1964) and Pao (1965) have proposed the following form for $F(\kappa)$, which we will use for estimation purposes:

$$F(\kappa) = n\chi\epsilon^{-\frac{1}{3}}\kappa^{-\frac{5}{3}}\exp\left(-\tfrac{3}{2}nD\epsilon^{-\frac{1}{3}}\kappa^{\frac{4}{3}}\right). \tag{16}$$

n is an adjustable constant, and χ and ϵ are the rates of molecular dissipation of temperature variance and turbulent kinetic energy; they can be written as

$$\left.\begin{aligned}\chi &= 6D\,\overline{(\partial\theta/\partial x_1)^2},\\ \epsilon &= 15\nu\,\overline{(\partial u_1/\partial x_1)^2}.\end{aligned}\right\} \tag{17}$$

While using this form for $F(\kappa)$ to estimate the third-moment error according to (14), we can also use it to check the magnitude of

$$\overline{\frac{\partial\theta}{\partial x_1}\frac{\partial\theta}{\partial x_1}\frac{\partial u_1}{\partial x_1}}.$$

From (10) and (14), we have

$$\overline{\frac{\partial\theta}{\partial x_1}\frac{\partial\theta}{\partial x_1}\frac{\partial u_1}{\partial x_1}} = \frac{-D}{7{\cdot}5}\int_0^{\infty}\kappa^4F(\kappa)\,d\kappa. \tag{18}$$

Carrying out the integration,

$$\left.\begin{aligned}\overline{\left(\frac{\partial\theta}{\partial x_1}\right)^{3^m}} &= 0{\cdot}145c\chi\epsilon^{\frac{1}{2}}D^{-\frac{3}{2}}n^{-\frac{3}{2}},\\ \overline{\frac{\partial\theta}{\partial x_1}\frac{\partial\theta}{\partial x_1}\frac{\partial u_1}{\partial x_1}} &= -0{\cdot}048\chi\epsilon^{\frac{1}{2}}D^{-\frac{3}{2}}n^{-\frac{3}{2}}.\end{aligned}\right\} \tag{19}$$

At this point we need a value for n. Because of the κ^4 in the integrands of (14) and (18), the results depend most strongly on the spectral form in the dissipative range; ideally then n would be chosen to make at least that portion of our assumed spectrum agree with observations. This does not appear possible at present for two reasons. First, $F(\kappa)$ is not directly measureable. It can, however, be related to the one-dimensional spectrum $F_1(\kappa_1)$, which can be measured,

$$F_1(\kappa_1) = \int_{\kappa_1}^{\infty} \frac{F(\kappa)}{\kappa} d\kappa. \tag{20}$$

Second, most of the published measurements of $F_1(\kappa_1)$ are confined to wave-numbers below the dissipative range. In that region, (16) and (20) give

$$F_1(\kappa_1) = \tfrac{3}{5} n \chi \epsilon^{-\frac{1}{3}} \kappa_1^{-\frac{5}{3}}. \tag{21}$$

Available data indicate, although not conclusively, that in (21) $n \simeq 0{\cdot}6$ and we will base our estimates on this value.

In air, our estimates (19) become

$$\left.\begin{aligned} \overline{\left(\frac{\partial \theta}{\partial x_1}\right)^{3m}} &= 6{\cdot}15c \overline{\left(\frac{\partial \theta}{\partial x_1}\right)^2} \left[\overline{\left(\frac{\partial u_1}{\partial x_1}\right)^2}\right]^{\frac{1}{2}}, \\ \overline{\frac{\partial \theta}{\partial x_1}\frac{\partial \theta}{\partial x_1}\frac{\partial u_1}{\partial x_1}} &= -2{\cdot}05 \overline{\left(\frac{\partial \theta}{\partial x_1}\right)^2} \left[\overline{\left(\frac{\partial u_1}{\partial x_1}\right)^2}\right]^{\frac{1}{2}}. \end{aligned}\right\} \tag{22}$$

The second prediction should not be too difficult to check experimentally. To use the first, we must find an expression for the velocity sensitivity c.

Heat transfer from a long thin wire in forced convection is expressed by

$$I^2 R - H(\Theta_w - \Theta) = C \frac{\partial \Theta_w}{\partial t}, \tag{23}$$

where I is the wire current, R its resistance, C its heat capacity, H is the convective heat transfer coefficient, and Θ and Θ_w are ambient fluid and wire temperatures. In constant-current operation in turbulence, R, H, and Θ vary with time, and we break them into mean and fluctuating parts:

$$\left.\begin{aligned} R &= \bar{R} + r, & \Theta &= \bar{\Theta} + \theta, \\ H &= \bar{H} + h, & \Theta_w &= \bar{\Theta}_w + \theta_w. \end{aligned}\right\} \tag{24}$$

The linearized equation for the fluctuations, obtained by substituting (24) into (23), is

$$\theta_w + \frac{C}{\bar{H}} \frac{\partial \theta_w}{\partial t} = \theta - \frac{h}{\bar{H}} (\bar{\Theta}_w - \bar{\Theta}), \tag{25}$$

for small overheats. This equation verifies the earlier statement that the wire temperature responds not only to ambient temperature changes, but also to changes in the heat-transfer coefficient. It shows that the apparent or measured fluid temperature fluctuation is

$$\theta^m = \theta - (h/\bar{H})(\bar{\Theta}_w - \bar{\Theta}). \tag{26}$$

In order to relate $h/\overline{H}$ to the velocity sensitivity c, we use the results of Collis & Williams (1959), presented in the survey paper by Corrsin (1963, p. 524), for heat transfer from hot wires. They found for the range $0{\cdot}02 \leqslant Re \leqslant 44$ in air

$$\overline{H} = \pi lk(0{\cdot}24 + 0{\cdot}56\, Re^{0{\cdot}45}), \tag{27}$$

where Re is the Reynolds number based on wire diameter and flow speed normal to the wire, l is wire length, and k is the thermal conductivity of air. It follows from (27) that for small fluctuation levels

$$\frac{h}{\overline{H}} = \frac{u_1}{\overline{U}_1}\left\{\frac{0{\cdot}25\, Re^{0{\cdot}45}}{0{\cdot}24 + 0{\cdot}56\, Re^{0{\cdot}45}}\right\}. \tag{28}$$

From (1), (26) and (28) the expression for velocity sensitivity is, in terms of easily-measured quantities,

$$c = \frac{\overline{I}^2\overline{R}(0{\cdot}25\, Re^{0{\cdot}45})}{\pi kl\overline{U}_1(0{\cdot}24 + 0{\cdot}56\, Re^{0{\cdot}45})^2}. \tag{29}$$

3. The effect under typical atmospheric conditions

A calculation using parameters typical of recent atmospheric experiments (0·6 micron diameter platinum wire carrying 0·3 ma in a 5 m/sec air flow) shows the importance of velocity sensitivity. For this wire $R/l = 3150\,\Omega/\text{cm}$, and (29) gives $c = 0{\cdot}00032\,^\circ\text{C sec cm}^{-1}$. Direct experience with this wire in the velocity-sensing mode allows an independent estimate of c, without the use of (27); this gives a c value about 50 % less. The difference is probably due to uncertainties in both estimates, and we take the average, $c = 0{\cdot}00024\,^\circ\text{C sec cm}^{-1}$, as representative. We use values of temperature and velocity gradients from the study of Gibson *et al.* (1970) of atmospheric turbulence over the sea,

$$\overline{(\partial u_1/\partial x_1)^2} = 29\,\text{sec}^{-2}; \quad \overline{(\partial\theta/\partial x_1)^2} = 1{\cdot}8\times 10^{-4}\,^\circ\text{C}^2\,\text{cm}^{-2}. \tag{30}$$

Equation (3) shows that the second moment is hardly affected by the velocity sensitivity,

$$\overline{\left(\frac{\partial\theta}{\partial x_1}\right)^{2m}} = 1{\cdot}01\,\overline{\left(\frac{\partial\theta}{\partial x_1}\right)^{2}}. \tag{31}$$

However, (22) shows that the measured skewness, which in our isotropic model is entirely due to velocity sensitivity, is large:

$$\overline{\left(\frac{\partial\theta}{\partial x_1}\right)^{3m}} \Big/ \left[\overline{\left(\frac{\partial\theta}{\partial x_1}\right)^{2m}}\right]^{\frac{3}{2}} \simeq 0{\cdot}6. \tag{32}$$

The values given by Stewart (1969) and Gibson *et al.* (1970) for skewness measured with fine resistance wires in the atmosphere are in the range 0·4 to 1·0, and have been cited as evidence that high Reynolds number turbulence need not be even locally isotropic. The present results suggest that these large skewnesses *could* be the result of velocity sensitivity contamininating the measurement of a locally isotropic temperature field. On the other hand, the temperature derivative field *could* be skew, and therefore anisotropic; in this case the relevance of the present isotropic calculations is not clear. It appears that more data, in which the effects of velocity sensitivity are carefully accounted for, are needed to settle this question.

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